11) $\int \frac{x^2-4}{x}\,dx$

$\int\left(x - \frac{4}{x}\right)dx$

$\frac{1}{2}x^2 - 4\ln|x| + C$

$\frac{1}{2}x^2 - \ln x^4 + C$

13) $\frac{1}{3}\int \frac{3(x^2+2x+3)}{x^3+3x^2+9x}\,dx$

$u = x^3+3x^2+9x$

$du = (3x^2+6x+9)\,dx$

$\frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|x^3+3x^2+9x| + C$

19) $\int \frac{x^4+x-4}{x^2+2}\,dx = \int\left(x^2-2+\frac{1}{2}\int\frac{2x}{x^2+2}\right)dx$

$u = x^2+2$

$du = 2x\,dx$

$$\begin{array}{r} x^2 \quad -2 \quad + \frac{x}{x^2+2} \\ x^2+0x+2 \overline{)\, x^4+0x^3+0x^2+x-4} \\ -(x^4+0x^3+2x^2) \\ \hline -2x^2+x-4 \\ -2x^2+0x-4 \\ \hline x \quad 0 \end{array}$$

$= \frac{1}{3}x^3 - 2x + \frac{1}{2}\ln|x^2+2| + C$

$\boxed{= \frac{1}{3}x^3 - 2x + \ln\sqrt{x^2+2} + C}$

21) $\int \frac{(\ln x)^2}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$\int u^2\,du \quad \frac{1}{3}(\ln x)^3 + C$

25) $\int \frac{2x}{(x-1)^2}\,dx = \int \frac{2x}{x^2-2x+1}\,dx$

$u = x^2-2x+1$

$du = (2x-2)\,dx$

$\boxed{u+1 = x}$

$u = x-1$

$du = dx$

$\int \frac{2(u+1)}{u^2}\,du = \int \frac{2u+2}{u^2}\,du = \int \frac{2}{u}\,du + \int \frac{2}{u^2}\,du$

$= 2\ln|x-1| - \frac{2}{(x-1)}$

$\boxed{\ln(x-1)^2 - \frac{2}{x-1} + C}$

27) $\int \frac{1\sqrt{2x}}{(1+\sqrt{2x})\sqrt{2x}}\,dx$

$u - 1 = \sqrt{2x}$

$u = 1+\sqrt{2x}$

$du = \frac{1}{\sqrt{2x}}\,dx$

$\int \frac{u-1}{u}\,du$

Interesting problem. We needed the √2x on the bottom, so we multiplied the top and bottom both by √2x. Then we solved √2x in terms of u!

$\int\left(1 - \frac{1}{u}\right)du = u - \ln|u| + C$

$(1+\sqrt{2x}) - \ln|1+\sqrt{2x}| + C$

29) $2\int \frac{\sqrt{x}\sqrt{x}}{2\sqrt{x}(\sqrt{x}-3)}\,dx = 2\int \frac{(u+3)^2}{u}\,du$

multiply these to get x

$(u+3)^2 = x$

Solve for x

$u = \sqrt{x} - 3$

$du = \frac{1}{2\sqrt{x}}\,dx$

FOIL

$2\int \frac{u^2+6u+9}{u}\,du$

Split the fraction and
reduce the terms.

$2\int \left(u + 6 + \frac{9}{u}\right) du = 2\left[\frac{1}{2}u^2 + 6u + 9\ln|u|\right] + C$

$= (\sqrt{x}-3)^2 + 12(\sqrt{x}-3) + 18\ln|\sqrt{x}-3| + C$